Super-resolution Sparse Projected Capacitive Multitouch Sensing

Mehrdad Yaghoobi,
In collaboration with: Stephen McLaughlin, and Mike E. Davies

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Capacitive Multitouch Sensing

Multipoint touchscreen (US 7663607 B2)

US007663607B2

(12) **United States Patent**
Hotelling et al.

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(45) **Date of Patent: Feb. 16, 2010**

(54) **MULTIPOINT TOUCHSCREEN**

(75) Inventors: **Steve Hotelling**, San Jose, CA (US); **Joshua A. Strickon**, San Jose, CA (US); **Brian Q. Huppi**, San Francisco, CA (US)

(73) Assignee: **Apple Inc.**, Cupertino, CA (US)

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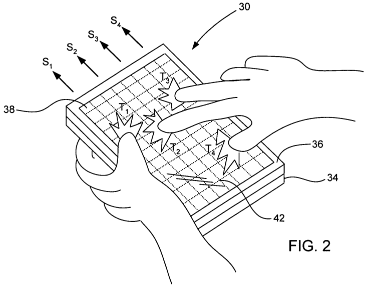


FIG. 2

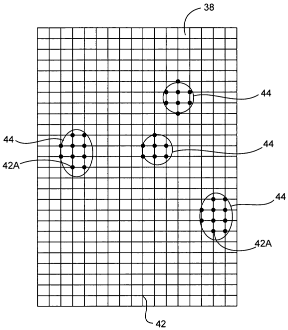
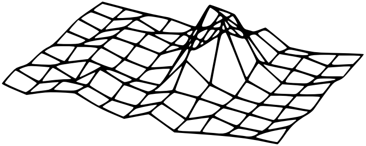
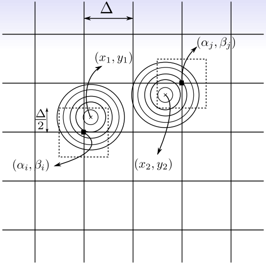


FIG. 3

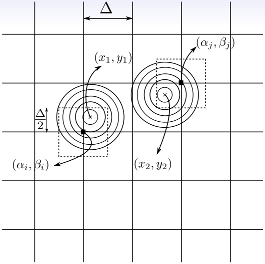
Multitouch Signal Modelling





$$
\begin{aligned}
t(x,y) &\approx \sum_{1 \le i \le K} a_i e^{-\frac{(x-x_i)^2+(y-y_i)^2}{\sigma^2}} \\
&= e^{-\frac{x^2+y^2}{\sigma^2}} * \sum_{1 \le i \le K} a_i \delta_{x_i, x_j}(x,y) \\
&= g(x,y) * d(x,y)
\end{aligned}
$$

Multitouch Signal Sampling



Uniform Sampling on a Fixed Grid

$$\mathbf{z}_{i,j} = \int_{\beta_i-\Delta/2}^{\beta_i+\Delta/2} \int_{\alpha_i-\Delta/2}^{\alpha_i+\Delta/2} \delta_{\alpha_i,\beta_i}(x,y)\, t(x,y)\, dx\, dy$$

$$\mathbf{z} = \mathcal{T}_g d(x,y), \ \mathbf{z} \in \mathbb{R}_+^{MN}$$

Touch Sensing and Super-resolution

- **Goal:** recovering touch locations, *i.e.* $d(x, y)$.
- **Challenges:**
  1. Streams of pulses are not band limited, *i.e.* Shannon-Nyquist reconstruction algorithm is not accurate .
  2. Resolution limitation: sampling happens on a coarse grid.
- **Resolution Enhancement Techniques:** (linear) interpolation with a triangle or a more complex kernel, *e.g.* Cubic Spline.
- **Proposed Resolution Enhancement Technique:** sparsity based single-exposure super-resolution

Super-resolution Touch Sensing using a Fine Virtual Grid

- **Aim:** a low-complexity algorithm.
- **Approach:** assuming that touches are happened on a fine virtual grid.
- **Properties:**
  1. A step towards to the ideal location recovery.
  2. No need to use complicated optimisation algorithms.
  3. Makes a good trade off between accuracy and noise sensitivity.

Sparsity Based Super-resolution

- A set of normalised elementary Gaussian signals, with centres on the virtual grid, is generated, *i.e.* Dictionary $\mathbf{D} = [\mathbf{d}_i]_{i \in \mathcal{J}}$,

$$\mathbf{d}_i = \gamma g(x - \hat{\alpha}_i, y - \hat{\beta}_i).$$

- Sparse signal approximation:

$$\min_{\theta \in \mathcal{C}} \|\mathbf{z} - \mathbf{D}\theta\|_2^2, \text{ s. t. } \|\theta\|_0 \leq K$$

- Assumptions:
  1. Centres of touches can not become very close, *i.e.* admissible set $\mathcal{C}$ and touches are R-apart.
  2. At most K touches are happened at the same time.
  3. θ is positive.
  4. The noise is zero-mean Gaussian.

Sparse Reconstruction Algorithms

- Proposed Model Based Sparse Approximation Algorithms:
  1. **Support Constrained Positive Hard Thresholding (SCPHT):**

$$
\begin{array}{l}
\theta_t \leftarrow \mathbf{D}^T\mathbf{z} \\
\textbf{while } k \leq K \textbf{ do} \\
\quad s_k \leftarrow \mathrm{argmax}_s\, \theta_t(s) \\
\quad \theta(s_k) \leftarrow \theta_t(s_k) \\
\quad \theta_t \leftarrow \mathcal{H}_{R/2}(\theta_t, s_k) \\
\quad k \leftarrow k + 1 \\
\textbf{end while}.
\end{array}
$$

    - Computational complexity $\mathcal{O}(P(\log(MN) + \log(P)))$.
    - It works better, when the dynamic range is small.

Sparse Reconstruction Algorithms

2. **Support Constrained Positive Matching Pursuit (SCPMP):**

$\mathbf{r} \leftarrow \mathbf{z}$
$\theta_r \leftarrow \mathbf{D}^T \mathbf{r}$
while $k \leq K$, **do**
    $s_k \leftarrow \mathrm{argmax}_{s \in \mathrm{supp}(\mathbf{m})} \theta_r(s)$
    $\theta(s_k) \leftarrow \theta_r(s_k)$
    $\mathbf{m} \leftarrow \mathcal{H}_{R/2}(\mathbf{m}, s_k)$
    $\mathbf{r} \leftarrow \mathbf{r} - \mathbf{d}_{s_k}\theta(s_k)$
    $\theta_r \leftarrow \theta_r - \mathbf{D}^T \mathbf{d}_{s_k}\theta(s_k)$
    $k \leftarrow k + 1$
end while

- Computational complexity $\mathcal{O}(K(2P + MN) + P\log(MN))$.
- More complex than SCPHT, but it performs better, when dynamic range and noise are large.

Theoretical Analysis of Recovery

Definition (Conditional Coherence)

The conditional coherence of a normalised dictionary, is defined as,

$$\mu_R = \sup_{(i,j)\in\mathcal{C}_R} |\langle \mathbf{d}_i, \mathbf{d}_j \rangle|$$

where, $\mathcal{C}_R$ is an admissible set.

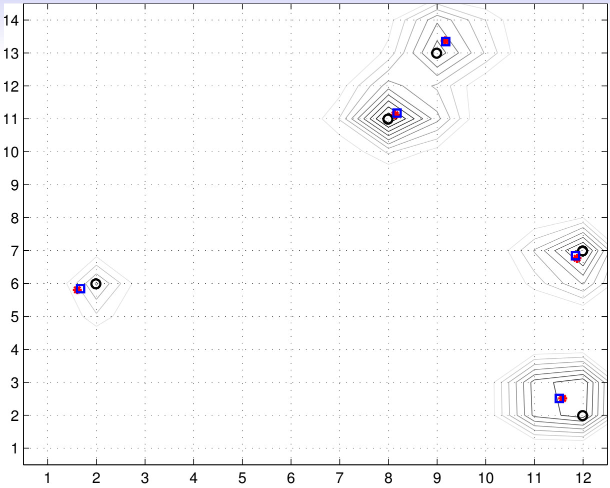
Theorem

For a positive $\rho \leq R/4$, a sufficient condition for the K sparse $\tilde{\theta}$, found by SCPMP or SCPHT, to have the atom index set in ρ neighbourhood of the original locations, is,

$$(\alpha\mu_{R/2}(K-1) + \mu_\rho)\frac{\theta_{max}}{\theta_{min}} + \beta\frac{\|\mathbf{e}\|_2^2}{\theta_{min}} < 1,$$

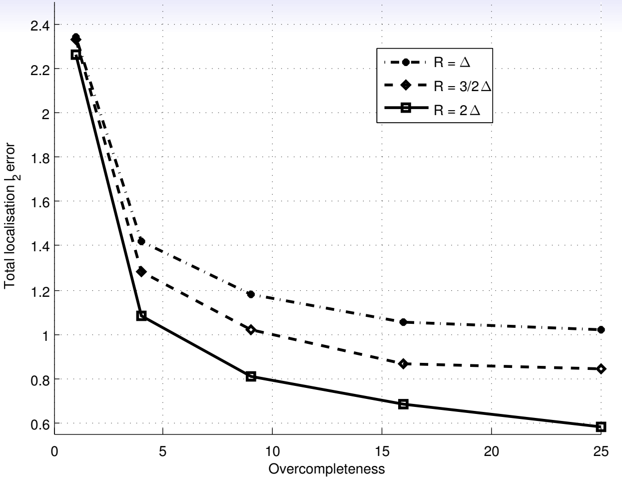
where α and β are some algorithm dependent constants.

Simulations: Recovery by SCPMP



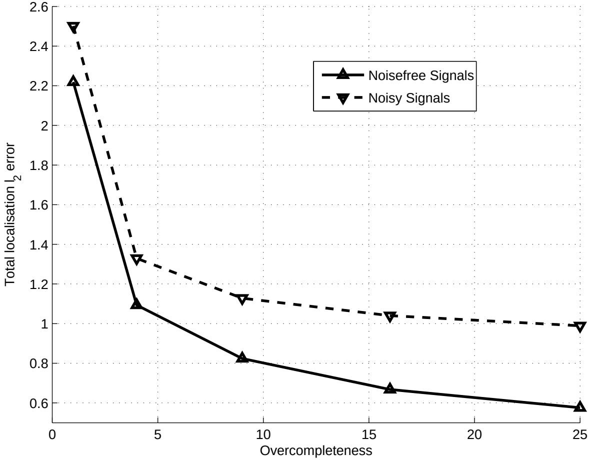
Stars: original locations. Circles: complete dictionary. Squares: 36 times overcomplete.

Simulations: Average Recovery by SCPMP



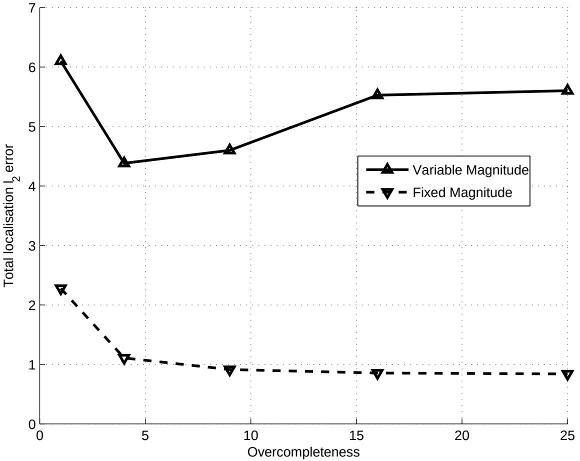
Δ: Pitch spacing, R: minimum separation, # of touches: 5 and 1000 trials.

Simulations: Noisy Signals



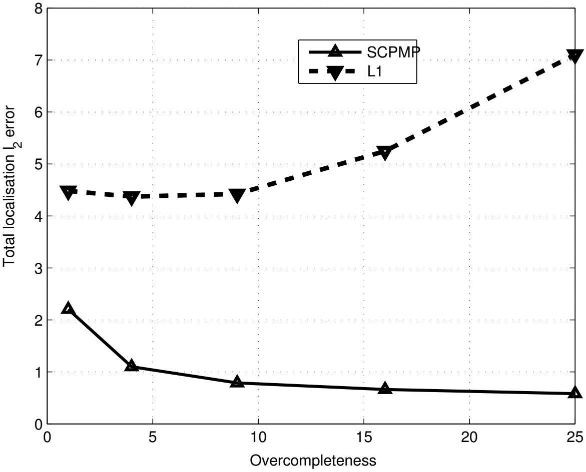
$0.5 \leq a_i \leq 1$, $\sigma = .1$

Simulations: Average Signal Recovery by SCPHT



Variable: $0.5 \leq a_i \leq 1$, Fixed: $a_i = 1$

Simulations: Comparison with the ℓ_1 Super-resolution



Basis Pursuit *v.s.* SCPMP

Conclusion

- Multitouch super-resolution problem was formulated as a sampling problem.
- A framework for the touch location recovery was proposed.
- Two low-complexity recovery algorithms were proposed.
- The benefits of using such a super-resolution technique was demonstrated by some simulations.
- The framework is robust to the observation noise and model mismatch.

Thanks for your attention.